Math 3

5.5 Graphing Rational Expressions

Unit 5

SWBAT graph rational expressions, state points of discontinuity, and find any horizontal or vertical asymptotes.

Example 1: Simplify the following. State any restrictions on the variables.

a) $\frac{(x+1)(x-5)}{(x-5)(x^2-1)} = \frac{(x+1)(x-5)}{(x-5)(x+1)(x-1)} = \frac{1}{x-1}$

$x \neq 5, \pm 1$

b) $\frac{x^2+x-12}{x^2+7x+12}$ $\frac{(x+4)(x-3)}{(x+4)(x+3)} = \frac{x-3}{x+3}$

$x \neq -4, -3$

x-int are any quantities that are left in the numerator

Vertical Asymptotes: Where the denominator of a function equals zero.

Point of Discontinuity: A hole in the graph.

Example 2: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2-1}{x^2-6x+5}$.

$\frac{(x-1)(x+1)}{(x-2)(x-3)}$

Holes: None

VA: $x = 2, 3$

x-int: $(0,1)$ & $(0,-1)$

Example 3: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2-4}{x^2+5x+6}$

$\frac{(x-2)(x+2)}{(x+2)(x+3)}$

Holes: $x = -2$

VA: $x = -3$

x-int: $(0,2)$

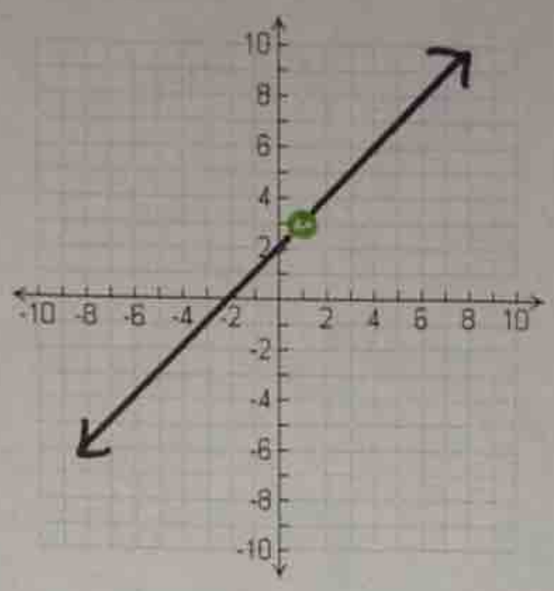
Horizontal Asymptotes: determined by comparing the degree of the numerator to the degree of the denominator. Let ***m*** = degree of numerator and ***n*** = degree of denominator.

If...	Then the graph has...
$m < n$ (larger degree in denominator) $f(x) = \frac{x+4}{x^2+5x+4}$ $(x+4)(x+1)$ $y=0$	A horizontal asymptote at y = 0 V.A.: $x = -1$ Hole(s): $x = -4$ x-int: None H.A.: $y = 0$ Domain: $(-\infty,-4) \cup (-4,-1) \cup (-1,\infty)$
$m = n$ (equal degrees & the fraction is the leading coefficients) H.A. $\neq 0$ $(x+4)(x+1)$ $f(x) = \frac{x^2+5x+4}{4x^2-9}$ $(2x-3)(2x+3)$	A horizontal asymptote at the coefficient of m divided by the coefficient of n V.A.: $x = -3/2, 3/2$ Hole(s): None x-int: $(-4,0)$ $(-1,$ H.A.: $y = \frac{1}{4}$ Domain: $(-\infty, -3/2) \cup (-3/2, 3/2) \cup (3/2, \infty)$
$m > n$ (larger degree in numerator) no H.A. $\frac{(x+4)(x+1)}{x+4}$ $f(x) = \frac{x^2+5x+4}{x+4}$	No horizontal asymptote V.A.: None Hole(s): $x = -4$ x-int: $(-1,0)$ H.A.: None Domain: $(-\infty,-4) \cup (-4,\infty)$

Example 4: State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

a) $f(x)=\dfrac{x^2+x-2}{x-1}$

$\dfrac{(x+2)(x-1)}{x-1}$



Holes: $x=1$

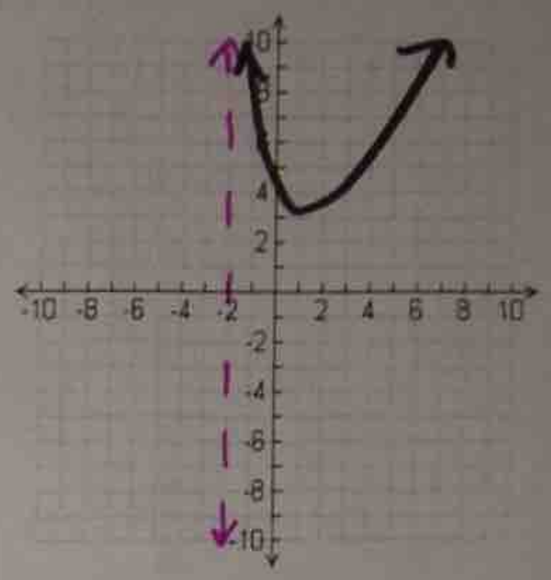
VA: None

HA: None

x-int: $(-2,0)$

Domain: $(-\infty,1)\cup(1,\infty)$

b) $f(x)=\dfrac{2x^2+3}{x+2}$



Holes: None

VA: $x=-2$

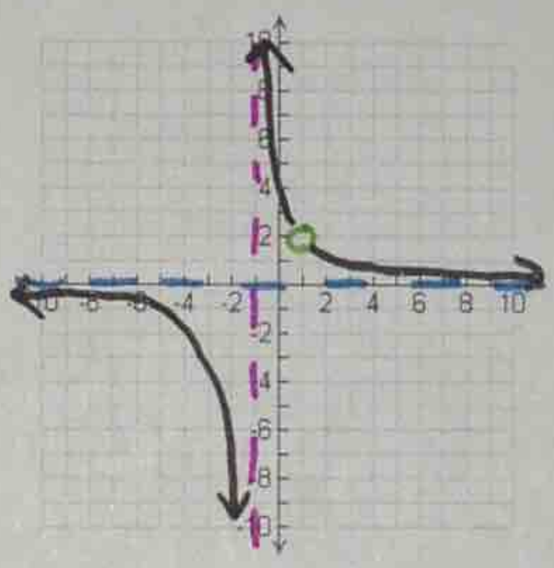
HA: None

x-int: None (imaginary)

Domain: $(-\infty,-2)\cup(-2,\infty)$

c) $f(x)=\dfrac{x-1}{x^2-1}$

$\dfrac{x-1}{(x-1)(x+1)}$



Holes: $x=1$

VA: $x=-1$

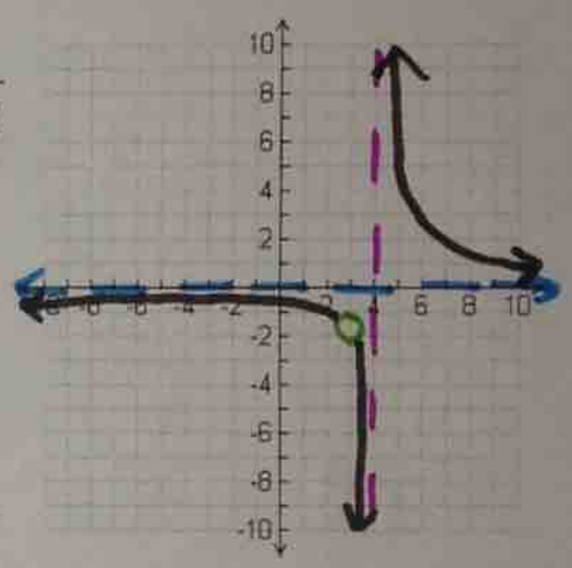
HA: $y=0$

x-int: None

Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

d) $f(x)=\dfrac{x-3}{x^2-7x+12}$

$\dfrac{x-3}{(x-4)(x-3)}=\dfrac{1}{x-4}$



Holes: $x=3$

VA: $x=4$

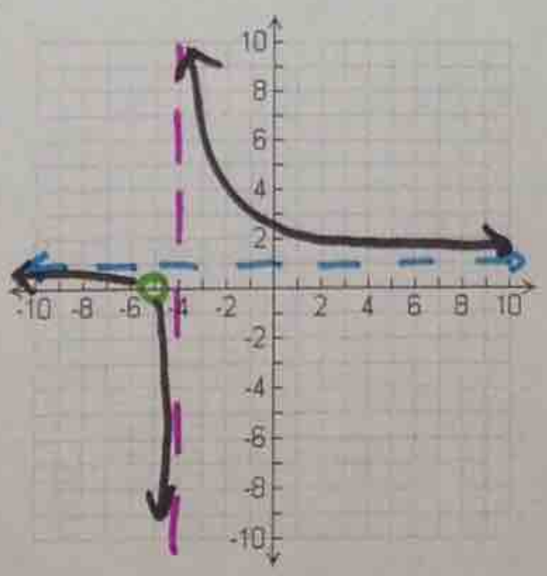
HA: $y=0$

x-int: None

Domain: $(-\infty,3)\cup(3,4)\cup(4,\infty)$

e) $f(x)=\dfrac{x^2+10x+25}{x^2+9x+20}$

$\dfrac{(x+5)(x+5)}{(x+5)(x+4)}$



POD: $x=-5$

VA: $x=-4$

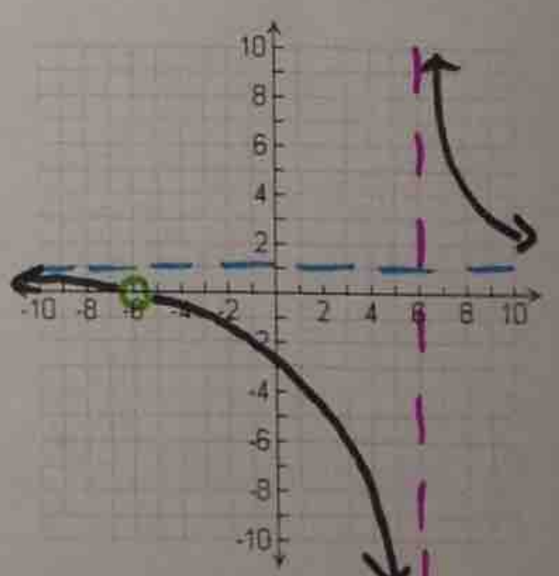
HA: $y=1$

x-int: $(-5,0)\to$ DNE

Domain: $(-\infty,-5)\cup(-5,-4)\cup(-4,\infty)$

f) $f(x)=\dfrac{x^2+12x+36}{x^2-36}$

$\dfrac{(x+6)(x+6)}{(x+6)(x-6)}$



POD: $x=-6$

VA: $x=6$

HA: $y=1$

x-int: $(-6,0)\to$ DNE

Domain: $(-\infty,-6)\cup(-6,6)\cup(6,\infty)$